Gauge Symmetry Breaking in High Dimensional Theory

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Summary

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I. INTRODUCTION

The SM has been confirmed by the current experiments at 100 GeV scale as the first approximation. However, there are some puzzles:

- The SM has three gauge groups with different gauge coupling constants. Why not one gauge group in the nature?
- Why do the particles in one family not form one representation? One step further, why are there three families? Why do all the particles not form one representation?
- Charge quantization.
- Gauge Hierarchy problem
- The SM does not include the gravity
-

Grand Unified Theory can solve some of the puzzles. However, there exist the new problems in the 4D scenarios where the GUT is broken by the Higgs mechanism:

- Proton decay problem (High unification scale in MSSM)
- Higgs doublet-triplet splitting problem (fine-tune)
- Fermion mass problem: GUT relation $m_e/m_\mu \quad m_d/m_s$ (add 45-plet Higgs for $SU(5)$)
-

These problems in 4D GUT can be solved naturally in the scenarios where the GUT gauge symmetry is broken by the discrete symmetry on the extra space manifold.

II. Principle

T. Li, hep-th/0112255, Nucl. Phys. B633:83-96,2002.

Consider the (4+n)-dimensional space-time manifold $M^4 \times E$ with coordinates $(x^\mu, y^1, y^2, ..., y^n)$, and one global discrete symmetry Γ on E for simplicity.

The action of any element $\gamma_i \subset \Gamma$ on E is

$$\gamma_i : \quad (y^1, y^2, ..., y^n) \longrightarrow (\gamma_i y^1, \gamma_i y^2, ..., \gamma_i y^n) . \qquad (1)$$

The Lagrangian is invariant under the discrete symmetry, i. e., for any element $\gamma_i \subset \Gamma$, we have

$$\mathcal{L}(x^\mu, \gamma_i y^1, \gamma_i y^2, ..., \gamma_i y^n) = \mathcal{L}(x^\mu, y^1, y^2, ..., y^n) \qquad (2)$$

So, for a generic bulk multiplet Φ which fills a representation of the bulk gauge group G, we have

$$\Phi(x^\mu, \gamma_i y^1, \gamma_i y^2, ..., \gamma_i y^n) = \eta_\Phi (R_{\gamma_i})^{l_\Phi} \Phi(x^\mu, y^1, y^2, ..., y^n) (R_{\gamma_i}^{-1})^{m_\Phi} , \qquad (3)$$

where R_{γ_i} is an element in G.

The consistency condition for R_{γ_i} is

$$R_{\gamma_i} R_{\gamma_j} = R_{\gamma_i \gamma_j} , \ \forall \gamma_i, \ \gamma_j \subset \Gamma$$

Mathematically speaking, the map $R : \ \Gamma \longrightarrow R_\Gamma \subset G$ is a homomorphism.

Definition. Suppose G is a Lie group and H is a subgroup of G, we denote the commutant of H in G as G/H i.e.

$$G/H \equiv \{g \in G | gh = hg, \text{ for any } h \in H\} .$$

Theorem I. For the zero modes, the gauge symmetry is G/R_Γ.

Proof. For the gauge field $A = A^a_\mu T^a$, $\eta_A = 0$. So, for a fixed gauge field $A^a_\mu T^a$, we have

$$A^a_\mu(x^\mu, \gamma_i y^1, \gamma_i y^2, ..., \gamma_i y^n) T^a = A^a_\mu(x^\mu, y^1, y^2, ..., y^n) R_{\gamma_i} T^a (R^{-1}_{\gamma_i}) .$$

In addition, if $A^a_\mu T^a$ has zero mode, it satisfies the equation

$$A^a_\mu(x^\mu, \gamma_i y^1, \gamma_i y^2, ..., \gamma_i y^n) T^a = A^a_\mu(x^\mu, y^1, y^2, ..., y^n) T^a .$$

Then, we obtain that $A^a_\mu T^a$ has zero mode if and only if $[R_{\gamma_i}, T^a] = 0$ ... R_{γ_i} and T^a are commute. So, under the discrete symmetry Γ, the gauge symmetry is G/R_Γ for th

$$
\begin{tikzcd}
U\psi(x,y) \arrow[r, "r_i"] & R_{r_i}U\psi(x,y) \quad UR_{r_i}\psi(x,y) \\
\psi(x\;y) \arrow[u, "U"] \arrow[r, "r_i"] & \begin{array}{l}\psi(x, ry \\ \equiv R_{r_i}\psi(x\;y\end{array} \arrow[u, "U"']
\end{tikzcd}
$$

Diagram Commute $\Rightarrow$

$[R_{r_i} \quad U] \quad 0$

$\Rightarrow$ for the zero modes the gauge group es G/R_Γ

Theorem II. If $u = (y^1 = u^1, y^2 = u^2, ..., y^n = u^n)$ is the fixed point of Γ, the gauge symmetry at that fixed point is G/R_Γ for all the KK modes.

Proof. At the fixed point $u = (y^1 = u^1, y^2 \quad u^2, ..., y^n = u^n)$ of Γ, we have for all γ_i

$$A^a_\mu(x^\mu, \gamma_i u^1, \gamma_i u^2, .. , \gamma_i u^n) T^a = A^a_\mu(x^\mu, u^1, u^2, ..., u^n) T^a$$

This theorem can be proved similarly.

Relation to the Wilson line gauge symmetry breaking. Assume that Γ is a global symmetry and acts freely on E, we can define the quotient manifold $B = E/\Gamma$. Because Γ belongs to the fundamental group of B,

$$\Gamma \subseteq \pi_1(B) ,$$

we obtain the Wilson line gauge symmetry breaking for Γ on B

Comments. For the standard gauge symmetry breaking by discrete symmetry, the rank of bulk gauge group can not be reduced if the discrete symmetry is Abelian, because R_Γ commutes with the Cartan subalgebra which is the maximal Abelian subalgebra of group G

III. Application. LEGUT

Grand Unified Theory:

- Gauge Coupling Unification.
- Fermion Unification. For example, for $SO(10)$, one family forms a 16 representation
- Problem from fermion unification: proton decay, D-T splitting, and fermion mass problem
-

We might conjecture that:

The realistic fundamental theory, which describes the nature, might be the theory with the gauge unification and without the fermion unification

In gauge unification theory, $M_U \sim O(100\mathrm{TeV})$, so it is very interesting if the string scale or the high dimensional Planck scale is at order of 100 TeVs.

Accelerated Gauge Coupling Unification

In 4D GUT, the runnings of the gauge couplings are

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M) + \frac{b_i}{4\pi} ln \frac{\mu^2}{M^2},$$

If there were N copies of gauge and Higgs sectors not far from the EW scale, we then have

$$\alpha_i^{-1}(\mu) - \alpha_j^{-1}(\mu) = \alpha_i^{-1}(M) - \alpha_j^{-1}(M) + N \frac{b_i - b_j}{4\pi} ln \frac{\mu^2}{M^2}.$$

The unification scale can be lowed in this approach, for instance, M_U can be around tens of TeV if $N = 13$

N. Arkani-Hamed, A. Cohen and H. Georgi, hep-th/0108089.

Similar results hold for 5D gauge theory, and the relative runnings are

$$\begin{aligned}\alpha_i^{-1}(\mu) - \alpha_j^{-1}(\mu) \quad & \quad \alpha_i^{-1}(M_c) - \alpha_j^{-1}(M_c) + \frac{b_i^0 - b_j^0}{4\pi} ln \frac{\mu^2}{M_c^2} \\ & + \sum_n \frac{b_i^n - b_j^n}{4\pi} ln \frac{\mu^2}{M_n^2}. \end{aligned} \tag{12}$$

For the simple case $b_i^n - b_j^n = b_i - b_j$, we have

$$\begin{aligned}\alpha_i^{-1}(\mu) - \alpha_j^{-1}(\mu) &= \alpha_i^{-1}(M_c) - \alpha_j^{-1}(M_c) + \frac{b_i^0 - b_j^0}{4\pi} ln \frac{\mu^2}{M_c^2} \\ & + \frac{b_i - b_j}{4\pi} ln \prod_n \frac{\mu^2}{M_n^2}, \end{aligned} \tag{13}$$

So, the unification scale M_U is about one order of magnitude larger than the compactification scale M_c due to the power running. So, $M_U = O(100\mathrm{TeV})$ if $M_c = O(10\mathrm{TeV})$

K. R. Dienes, E. Dudas, T. Gherghetta, hep-ph/9803466, hep-ph/9806292.

The beta functions for the MSSM are $(b_1', b_2', b_3') = (-33/5, -1, 3)$ (or $(b_2' - b_1', b_3' - b_2') = (28/5, 4)$). So, we require that

$$\begin{matrix} b_2 - b_1 \\ b_3 - b_2 \end{matrix} \sim (28/5)\ \ 4 \uparrow\ \ 1\ 4\ . \tag{14}$$

Non-Supersymmetric $SU(5)$ Model on $M^4 \times S^1/Z_2 \times S^1/Z_2$

T. Li, hep-th/0107136, Phys. Lett. B520:377-384, 2001; T. Li and W. Liao, hep-th/0207126.

We assume that the coordinates are x^μ, ($\mu = 0, 1, 2, 3$), $y \equiv x^5$ and $z \equiv x^6$. The radii for the circles along y direction and z direction are R_1 and R_2, respectively. The orbifold $S^1/Z_2 \times S^1/Z_2$ is obtained by $S^1 \times S^1$ modulo the equivalent classes:

$$P_y : \; y \sim -y \; , \; P_z : \; z \sim -z \; . \tag{15}$$

The four fixed points are $(y = 0, z = 0)$, $(y = 0, z = \pi R_2)$, $(y = \pi R_1, z = 0)$, and $(y = \pi R_1, z = \pi R_2)$. And the four fixed lines are $y = 0$, $y = \pi R_1$, $z = 0$, $z = \pi R_2$.

For a generic bulk field $\phi(x^\mu, y, z)$, we can define two parity operators P_y and P_z for the two reflective Z_2 symmetries, respectively

$$\phi(x^\mu, y, z) \; \rightarrow \phi(x^\mu, -y, z) = P_y \phi(x^\mu, y, z) \; , \tag{16}$$

$$\phi(x^\mu, y, z) \; \rightarrow \phi(x^\mu, y, -z) = P_z \phi(x^\mu, y, z) \tag{17}$$

Denoting the field $\phi(x^\mu, y, z)$ with (P_y, P_z)=$(\pm, \pm)$ by $\phi_{\pm\pm}$, we can calculate the KK mode expansions and the masses for KK modes. Properties of the bulk fields

1 Only ϕ_{++} fields have zero modes.

Only ϕ_{++} and ϕ_{+-} have non-zero values on the 4-branes at $y = 0$ and $y = \pi R_1$,

Only ϕ_{++} and ϕ_{-+} have non-zero values on the 4-branes at $z = 0$ and $z = \pi R_2$.

MODEL. We put three pair of Higgs 5-plets in the bulk H_u, H_d, H_u^2, H_d^2, H_u^3, H_d^3. For simplicity, we define $H_1 \equiv H_u^2$, $H_2 \equiv H_d^2$, $H_3 \equiv H_u^3$, $H_4 \equiv H_u^4$. For H_u and H_d, we choose $\eta_\phi^{p_y} = +1$ and $\eta_\phi^{P_z} = +1$. For H_i, we choose $\eta_\phi^{p_y} = -1$ and $\eta_\phi^{P_z} = +1$. In addition, we put the SM fermions on any fixed 3-brane, for example, the 3-brane at $(y = 0, z = 0)$.

Table 1: Parity assignment and masses ($n \geq 0, m \geq 0$) of the fields in the SU(5) gauge and Higgs multiplets for the non-supersymmetric model.

(P, P')	field	mass
$(+,+)$	A^a_μ, H^D_u, H^D_d	$\sqrt{n^2/R_1^2 + m^2/R_2^2}$
$(+,-)$	$A^{\hat{a}}_5$, A^a_6, H^T_i	$\sqrt{n^2/R_1^2 + (m+1)^2/R_2^2}$
$(-,+)$	A^a_5, $A^{\hat{a}}_6$, H^D_i	$\sqrt{(n+1)^2/R_1^2 + m^2/R_2^2}$
$(-,-)$	$A^{\hat{a}}_\mu$, H^T_u, H^T_d	$\sqrt{(n+1)^2/R_1^2 + (m+1)^2/R_2^2}$

Table 2. The gauge fields, Higgs fields and gauge group on the 3-branes that are located at the fixed points (y 0 z 0) (y 0 z πR_2), (y πR_1 z 0) and (y πR_1 z πR_2), and on the 4 branes that are located at y 0, 0, y πR_1 and z πR_2.

Brane position	field	Group
$(y = 0, z = 0)$	A^a_μ, H^D_u, H^D_d	G_{SM}
$(y = 0, z = \pi R_2)$	A^a_μ, H^D_u, H^D_d	G_{SM}
$(y = \pi R_1, z = 0)$	A^a_μ, H^D_u, H^D_d	G_{SM}
$(y = \pi R_1, z = \pi R_2)$	A^a_μ, H^D_u, H^D_d	G_{SM}
$y = 0$ or $y = \pi R_1$	A^a_μ, $A^{\hat{a}}_5$, A^a_6, H^D_u, H^D_d, H^T_i	G_{SM}
$z = 0$ or $z = \pi R_2$	A^a_μ, A^a_5, $A^{\hat{a}}_6$, H^D_u, H^D_d, H^D_i	G_{SM}

Proton Decay. From Table 2, we know that on the observable 3-brane at any one of the four fixed points, there exist only the SM gauge fields and one pair of Higgs doublets H_u and H_d. There are no operators which can lead to the proton decay.

Charge Quantization. The charge quantization is obtained due to the gauge invariance of the Yukawa couplings and anomaly cancellation

Gauge Coupling Unification. We consider the scenario with $R_1 \sim 10R_2$ and make the relative runnings to be at leading order. At the energy scale $1/R_1 < \mu < 1/R_2$ in the effective 5D theory, we have the SM gauge fields, adjoint scalar fields A_5^a, lepton-quark scalar $A_6^{\hat{a}}$ with hypercharge $5/6$ and three pairs of two Higgs doublets, and then, the beta functions are

$$b_1, b_2, b_3) \qquad \frac{43}{30}, \frac{11}{2}, \frac{21}{2}), \ (b_2 - b_1, b_3 - b_2) = (104/15, 14/3)$$

Because $(b_2 - b_1)/(b_3 - b_2) = 1.49$, we do have the accelerated gauge coupling unification. For example, assuming $1/R_2 = 6/R_1$, the gauge coupling unification can be achieved at $\Lambda \approx 190$ TeV for $1/R_1 = 10$ TeV.

N=2 SUSY $SU(5)$ Model on
$M^4 \times S^1/(Z_2 \times Z_2') \times S^1/(Z_2 \times Z_2')$

Reference: T. Li, hep-ph/0108120, Nucl. Phys. B619:75-104, 2001; T. Li and W. Liao, hep-th/0207126.

The orbifold $S^1/(Z_2 \times Z_z') \times S^1/(Z_2 \times Z_2')$ is defined by $\times S^1$ moduloing the equivalent classes

$$P_y \quad y \sim -y, \;\; P_y' \,, \; y' \sim -y' \tag{18}$$

$$P_z \quad z \sim -z, \;\; P_z' \quad z' \sim -z' \tag{19}$$

where $y' = y - \pi R_1/2$ and $z' = z - \pi R_2/2$. The physical space is in the region: $0 \leq y \leq \pi R_1/2$ and $0 \leq z \leq \pi R_2/2$.

The 6-dimensional $N=2$ supersymmetric theory is anomaly-free in the bulk. In terms of 4-dimensional language, the 6-dimensional $N=2$ supersymmetric theory corresponds to the 4-dimensional $N=4$ supersymmetric theory. The 6D gauge multiplet can be decomposed to be one vector superfield, V, and three chiral superfields, Σ_5, Σ_6 and Φ in 4D.

From the bulk action, we obtain that under P_y and $P_{y'}$, the vector multiplets transform as:

$$V(x^\mu, -y, z) = P_y V(x^\mu, y, z)(P_y)^{-1}, \tag{20}$$

$$V(x^\mu, -y', z) = P'_y V(x^\mu, y', z)(P'_y)^{-1} \tag{21}$$

$$\Sigma_5(x^\mu, -y, z) = -P_y \Sigma_5(x^\mu, y, z)(P_y)^{-1}, \tag{22}$$

$$\Sigma_5(x^\mu, -y', z) = -P'_y \Sigma_5(x^\mu, y', z)(P'_y)^{-1}, \tag{23}$$

$$\Sigma_6(x^\mu, -y, z) = P_y \Sigma_6(x^\mu, y, z)(P_y)^{-1}, \tag{24}$$

$$\Sigma_6(x^\mu, -y', z) = P'_y \Sigma_6(x^\mu, y', z)(P'_y)^{-1}, \tag{25}$$

$$\Phi(x^\mu \; -y, z) = -P_y \Phi(x^\mu, y, z)(P_y)^{-1}, \tag{26}$$

$$\Phi(x^\mu, -y', z) = -P'_y \Phi(x^\mu, y', z)(P'_y)^{-1} \tag{27}$$

For P_z and P'_z, the vector multiplet transformations are similar to those under P_y and $P_{y'}$, , *i. e*, we just make the following transformations on the subscripts: $y \leftrightarrow z$ and $5 \leftrightarrow 6$.

We choose the following representations for (P_y, P'_y, P_z, P'_z):

$$(+1,+1,+1,+1,+1), \quad P'_y = (+1,+1,+1,-1,-1), \quad (28)$$

$$(+1,+1,+1,+1,+1) \quad P'_z = (+1,+1,+1,-1,-1). \quad (29)$$

MODEL. We put one pair of Higgs doublets (hypermultiplet) Φ_{H_u} and Φ_{H_d}, and one Higgs singlet Φ_S with $U(1)$ charge 1 on the boundary 4-brane at $z = \pi R_2/2$, which preserves the 4-dimensional $N = 2$ supersymmetry. In order to avoid the proton decay, we put the quark fields (Q, U and D) on the 3-brane at $(y = \pi R_1/2, z = \pi R_2/2)$, and the lepton and neutrino fields (L, E, N) on the 3-brane at $(y = 0, z = \pi R_2/2)$

Anomaly Cancellation. Introducing the Chern-Simons term on the 4-brane at $z = \pi R_2/2$ or 6-dimensional topological term.

Table 3: Parity assignment and masses ($n \geq 0, m \geq 0$) for the vector multiplet in the supersymmetric $SU(5)$ model on $M^4 \times S^1/(Z_2 \times Z_z') \times S^1/(Z_2 \times Z_2')$. And we include the Higgs superfields (H, H^c) on the fixed 4-brane at $z = \pi R_2/2$.

$(P^y, P^{y'}, P^z, P^{z'})$	field	mass
$(+,+,+,+)$	V_μ^a	$\sqrt{(2n)^2/R_1^2 + (2m)^2/R_2^2}$
$(+,-,+,-)$	$V_\mu^{\hat{a}}$	$\sqrt{(2n+1)^2/R_1^2 + (2m+1)^2/R_2^2}$
$(-,-,+,+)$	Σ_5^a	$\sqrt{(2n+2)^2/R_1^2 + (2m)^2/R_2^2}$
$(-,+,+,-)$	$\Sigma_5^{\hat{a}}$	$\sqrt{(2n+1)^2/R_1^2 + (2m+1)^2/R_2^2}$
$(+,+,-,-)$	Σ_6^a	$\sqrt{(2n)^2/R_1^2 + (2m+2)^2/R_2^2}$
$(+,-,-,+)$	$\Sigma_6^{\hat{a}}$	$\sqrt{(2n+1)^2/R_1^2 + (2m+1)^2/R_2^2}$
$(-,-,-,-)$	Φ^a	$\sqrt{(2n+2)^2/R_1^2 + (2m+2)^2/R_2^2}$
$(-,+,-,+)$	$\Phi^{\hat{a}}$	$\sqrt{(2n+1)^2/R_1^2 + (2m+1)^2/R_2^2}$
$(P^y = +, P^{y'} = +)$	H_u, H_d, S	$2n/R_1$
$(P^y = -, P^{y'} = -)$	H_u^c, H_d^c, S^c	$(2n+2)/R_1$

Table 4: For the supersymmetric model $SU(5)$ on $M^4 \times S^1/(Z_2 \times Z_z') \times S^1/(Z_2 \times Z_2')$, the gauge superfields, the number of 4-dimensional supersymmetry and gauge symmetry on the 3-brane which is located at the fixed point $(y = 0, z = 0)$, $(y = 0, z = \pi R_2/2)$, $(y = \pi R_1/2, z = 0)$, and $(y = \pi R_1/2, z = \pi R_2/2)$, or on the 4-brane which is located at the fixed line $y = 0$, $z = 0$, $y = \pi R_1/2$, $z = \pi R_2/2$. We also include the fermions, left handed quark doublet Q, right handed up-type quark U and down-type quark D that are on the 3-brane at $(y = \pi R_1/2, z = \pi R_2/2)$, the lepton doublet L, right handed lepton E and neutrino N that are on the 3-brane at $(y = 0, z = \pi R_2/2)$.

Brane Position	Fields	SUSY	Group
$(0, 0)$	V_μ^A	$N = 1$	$SU(5)$
$(0, \pi R_2/2)$	V_μ^a, $\Sigma_6^{\hat{a}}$, L, E, N, H_u, H_d, S	N=1	G_{SM}
$(\pi R_1/2, 0)$	V_μ^a, $\Sigma_5^{\hat{a}}$	N=1	G_{SM}
$(\pi R_1/2, \pi R_2/2)$	V_μ^a, $\Phi^{\hat{a}}$, Q, U, D, H_u, H_d, S	N=1	G_{SM}
$y = 0$	V_μ^A, Σ_6^A	N=2	$SU(5)$
$z = 0$	V_μ^A, Σ_5^A	N=2	$SU(5)$
$y = \pi R_1/2$	V_μ^a, $\Sigma_5^{\hat{a}}$, Σ_6^a, $\Phi^{\hat{a}}$	N=2	G_{SM}
$z = \pi R_2/2$	V_μ^a, Σ_5^a, $\Sigma_6^{\hat{a}}$, $\Phi^{\hat{a}}$, Φ_{H_u}, Φ_{H_d}, Φ_S	N=2	G_{SM}

Charge Quantization. The charge quantization can be obtained due to: (1) gauge invariance of the localized superpotential $H_u D^c \Phi^{\hat{a}}$ on the 3-brane at $(y = \pi R_1/2, z = \pi R_2/2)$ and Yukawa superpotentials; (2) Anomaly cancellation.

Gauge Coupling Unification. We consider the scenario with $R_1 \sim 10 R_2$. At the energy scale $1/R_1 < \mu < 1/R_2$ in the effective 5D theory, we have two singlets fields with $U(1)_Y$ charge 1, four chiral Higgs doublet fields, V^a and Σ_5^a, and then, the beta functions are b_1, b_2, b_3 $6/5, 2, 6$ and $(b_2 - b_1, b_3 - b_2) =$ $16/5, 4)$. Because $(b_2 - b_1)/(b_3 - b_2) = 1.4$, we do have the accelerated gauge coupling unification. For example, assuming $1/R_2$ $20/R_1$ and $1/R_1$ 10 TeV, we can achieve the gauge coupling unification at around 530 TeV with the MSSM threshold scale M_{SUSY} about 200 1000 GeV.

IV. Non-Standard Gauge Unification Theory

A

(T. Li et al, in preparation)

<1> $SU(5)$ on $S^1/Z_2 \times S^1/Z_2$

$$U(1)_Y \sim \frac{1}{\alpha}\sqrt{5/3}\begin{bmatrix} -\frac{\alpha}{3} & & & & \\ & -\frac{\alpha}{3} & & & \\ & & -\frac{\alpha}{3} & & \\ & & & \frac{\alpha}{2} & \\ & & & & \frac{\alpha}{2} \end{bmatrix}$$

$(Q = T_3 + Q_Y)$

$|\alpha| \neq 1$, X, Y can not mediate the proton decay due to the $U(1)_Y$ Quantum Numbers

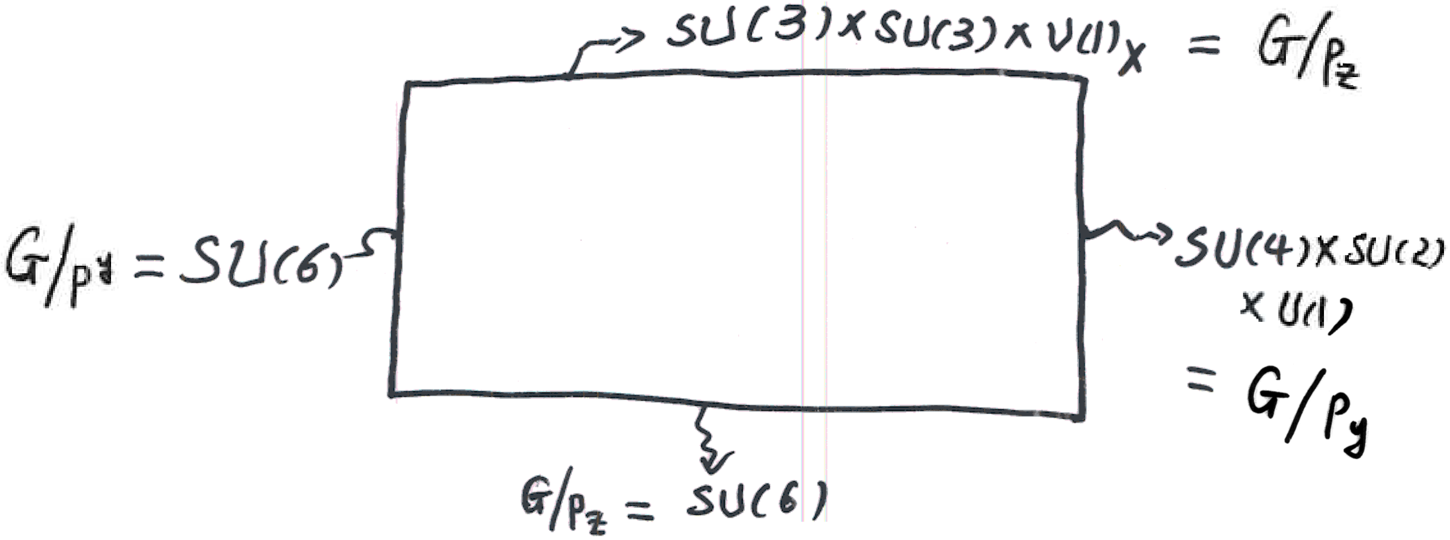
Conservation

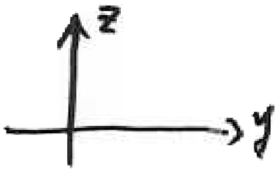
Question: Charge Quantization?

B

<2> N 2 SUSY SU(6 on

$M^4 \times S^1/Z_2 \times Z_2 \times S^1/(Z \times Z_2$





$= (+ \quad + \quad + \quad +, + \quad +)$

$- (+, +, + \quad +, + \quad +$

$- (+ \quad + \quad + \quad +, \quad -$

$(+ \quad + \quad + \quad - \quad -$

a c d e f $\equiv \begin{bmatrix} a & & & & & \\ & b & & & & \\ & & c & & & \\ & & & d & & \\ & & & & e & \\ & & & & & f \end{bmatrix}$

SU 3 x SU(3 model

T. L and W Liao hep ph/0202090

L J Hal and Y Nomura hep ph/0202107

S Dimopoulos D E Kaplan and N Wener hep ph/0202 35

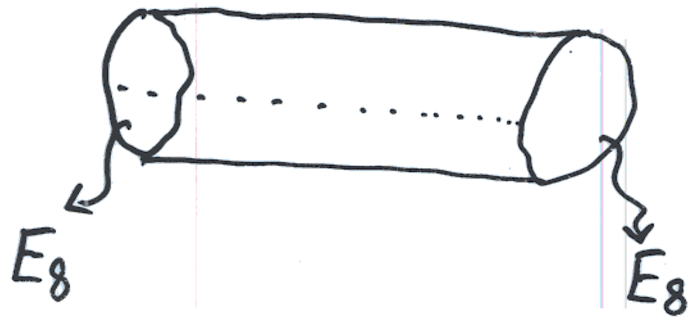
* Embed SU(3) x SU(3) model to above set up by putting SU(3 x SU 3) model on the 4 brane at $z = \pi R_2/2$
The SM fermions and Higgs have no charges under $U(1)_X$

Charge Quantization

EW mixing angles

V Model Building in M theory on S^1/Z_2

B Ovrut et a
in preparation



$$E_8 \xrightarrow{SU(5)} SU(5) \xrightarrow[\text{Wilson line}]{Z_2} SU(3) \times SU(2) \times U(1)$$

$$E_8 \xrightarrow{SU(4)} SO(\ 0) \xrightarrow{Z_2} \begin{cases} \text{Flipped } SU(5 \\ \text{Pati Salam Model} \end{cases}$$

$$E_8 \xrightarrow{SU(4)} SO(\ 0) \xrightarrow[\text{or } Z_6]{Z_2 \times Z_2} SU(3) \times SU(2) \times U(1)^2$$

Point we must introduce Wilson line to break the GUT gauge symmetry because the dimensions of Higgs are smaller than that of the adjoint representation

VI. Conclusion.

(I) Principle: Gauge symmetry G can be broken down to G/R_Γ by the discrete symmetry Γ on the extra space manifold. And the fixed point of Γ preserve only the gauge symmetry G/R_Γ.

(II) Application:

(1) Extra dimensional (orbifold) GUT model buildings.

(2) Model buildings in M-theory on S

Advantage. Solving the 4D GUT problems: proton decay, D-T splitting, fermion mass problem.